National Aeronautics and
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Reply to Attn of: ED24-80-17

TO: ED21/Mr. Jewell

FROM: ED24/Ms. Spanyer

SUBJECT: Results of Calculations from Theoretical Equations Pertaining to Pogo Transfer Functions

REF: Fenwick, J., SSME Pogo Equations, Rapidfax Received February 14, 1980

Introduction

The purpose of this memorandum is to verify the calculations of the pogo transfer function analysis that was initially performed by the SSME system contractor, Rocketdyne. The initial equations applied in this analysis were developed by Rocketdyne prior to obtaining hot firing pogo pulsing data and they were acquired from the reference. A secondary objective of this memorandum is to document these equations and their various dimensionless forms along with the results of the calculations.

The initial set of twelve equations obtained from Rocketdyne represent a linearization of pressure fluctuations from the input of the low pressure oxidizer turbopump (LPOTP) to the output at the main combustion chamber (MCC). These equations are linear nonhomogenous algebraic equations with variable coefficients in the form of the Laplace transform variable S. Furthermore, the equations contain parameters such as inertance, resistance, and compliance that can be varied to determine the sensitivity of these parameters.

Discussion

The pogo transfer function calculations delineated herein were accomplished through mathematical analysis of the most significant elements of the SSME pogo system. The elements that were considered, in addition to the symbols pertaining to the various quantities in the mathematical development, are shown on Figure 1. It can be seen from this figure that the most important components are the LPOTP, the accumulator, the low pressure oxidizer discharge duct, the high pressure oxidizer turbopump (HPOTP), the high pressure oxidizer duct, the low pressure oxidizer turbine drive line, and the MCC.

(NASA-TM-87422) RESULTS OF CALCULATIONS FROM THEORETICAL EQUATIONS PERTAINING TO POGO TRANSFER FUNCTIONS (NASA) 14 p

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For the purpose of verifying the Rocketdyne calculations, the transfer function of $\left|\frac{\Delta P_{os2}}{\Delta P_{os1}}\right|$ was selected as the primary quantity of concern because it delineates the effects of the accumulator and it can eventually be compared to measured data. From Figure 1, it is seen that this quantity is the absolute value of the ratio of the input to the HPOTP relative to the input to the LPOTP. The results of the calculations are shown on Figure 2 for five different cases that were to be verified. These cases and their parameters are:

Case I: The engine with no suppressor,

($L^*_{os} = 0.001$ psi - $\frac{sec^2}{lb}$, $C_{s2} = 0.001 \frac{lb}{psi}$, $R_a = 0.28$ psi - $\frac{sec}{lb}$, $L_a = 0.0011$ psi - $\frac{sec^2}{lb}$, $C_a = 0.0916 \frac{lb}{psi}$, and $\Delta\dot{W}_a$ was deleted from equations 8 and 9 of the appendices)

Case II: The engine with the ICD minimum compliance suppressor,

($L^*_{os} = 0.001$ psi - $\frac{sec^2}{lb}$, $C_{s2} = 0.001 \frac{lb}{psi}$, $R_a = 0.028$ psi - $\frac{sec}{lb}$, $L_a = 0.0011$ psi - $\frac{sec^2}{lb}$, $C_a = 0.045 \frac{lb}{psi}$)

Case III: The engine with gox suppressor (same values as Case II except $C_a = 0.0916 \frac{lb}{psi}$)

Case IV: The engine as in Case III with the suppressor resistance increased by a factor of 4,

($R_a = 0.112$ psi - $\frac{sec}{lb}$)

Case V: The engine as in Case III with the suppressor resistance increased by a factor of 10,

($R_a = 0.28$ psi - $\frac{sec}{lb}$)

As can be seen in Figure 2, the results shown here do verify the Rocketdyne calculations for all five cases. In this figure, the transfer function $\left|\frac{\Delta P_{os2}}{\Delta P_{os1}}\right|$ is plotted as a function of frequency over the range of 0 - 50 Hz.

Transfer functions pertaining to other elements in the lox system could have also been developed from this program and may be developed at a later time if necessary. However, it was not considered necessary to develop the other quantities to demonstrate the required verification.

The secondary objective of this memorandum, which is to document the equations, is accomplished in Appendices 1, 2, and 3. In Appendix 1 are the equations pertaining to the fluctuating flow through the lox system as obtained from Rocketdyne. In Appendix 2 is the nondimensional version of these equations which were developed to be implemented on the analog computer. In Appendix 3 is the further linearized set of equations that were programmed in matrix form to obtain the desired pogo frequency response function.

Summary

In summary it should be noted that three things were accomplished. First, the calculations of the pogo transfer function analysis developed by Rocketdyne were verified by MSFC for each of the five cases; the engine with no suppressors, the engine with the ICD minimum compliance suppressor, and the engine with the gox suppressor using three different values for the suppressor resistance. Second, the equations developed to calculate the fluctuating flow through the lox system were documented in their original and various dimensionless forms. Third, a computational scheme was developed for determining the transfer function across any element of the system.

If there are any questions pertaining to this verification please contact the undersigned at 453-3158 or Luke Schutzenhofer at 453-2475.

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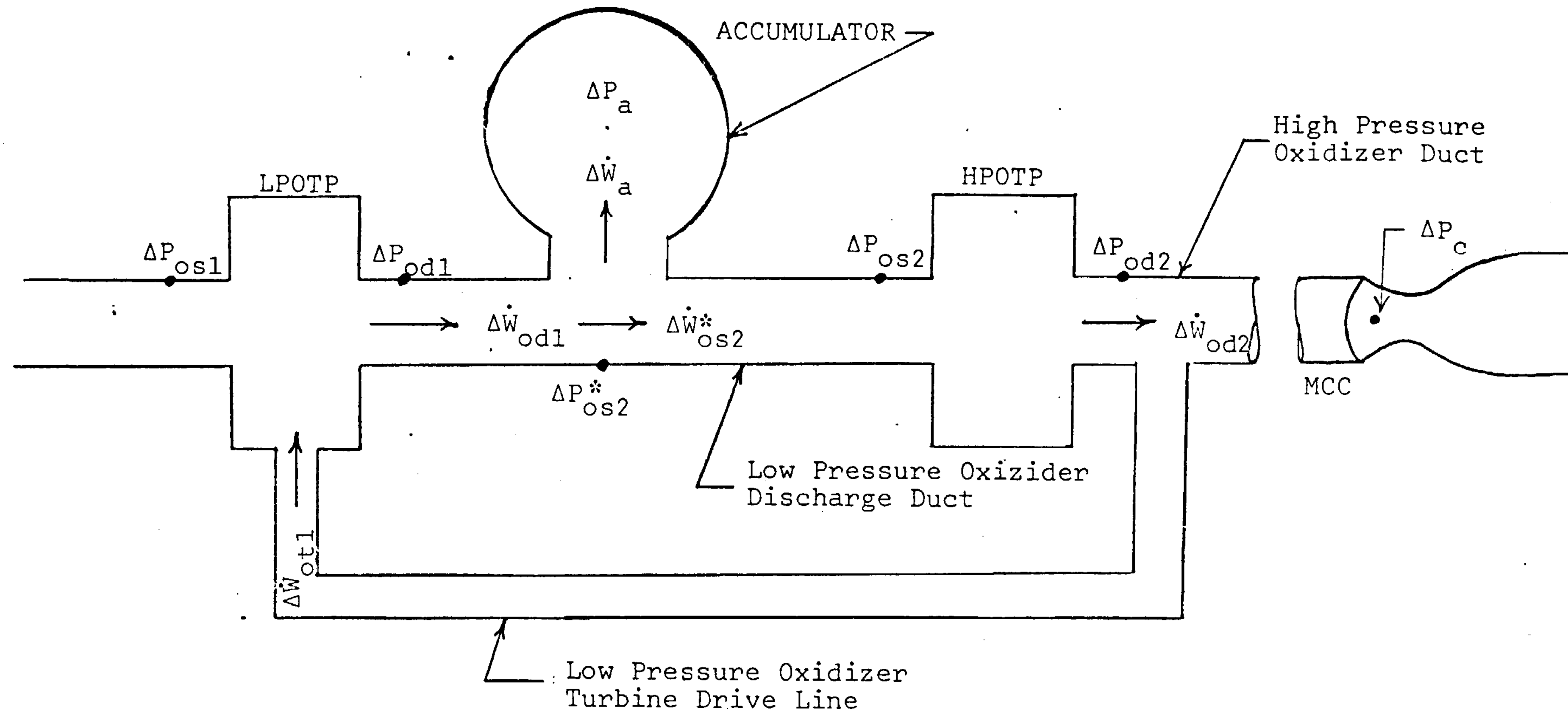
NOMENCLATURE FOR POGO ANALYSIS



<u>Symbols:</u>

ΔN_1 - Fluctuations in LPOP speed

ΔP - Pressure fluctuations

$\Delta \dot{W}$ - Flow rate fluctuations

L^*_{os} - Interpump duct inertance

C_a - Affective suppressor compliance

L_a - Affective suppressor inertance

R_a - Affective suppressor resistance

C_{s2} - High pressure lox pump inlet compliance

<u>Subscripts:</u>

o - Oxygen

s - Suction

d - Discharge

1 - LPOP

2 - HPOP

t - Turbine

a - Accumulator

c - Combustion chamber

FIGURE 1

ANALYTICAL ESTIMATE OF
LPOTP/INTERPUMP DUCT RESPONSE

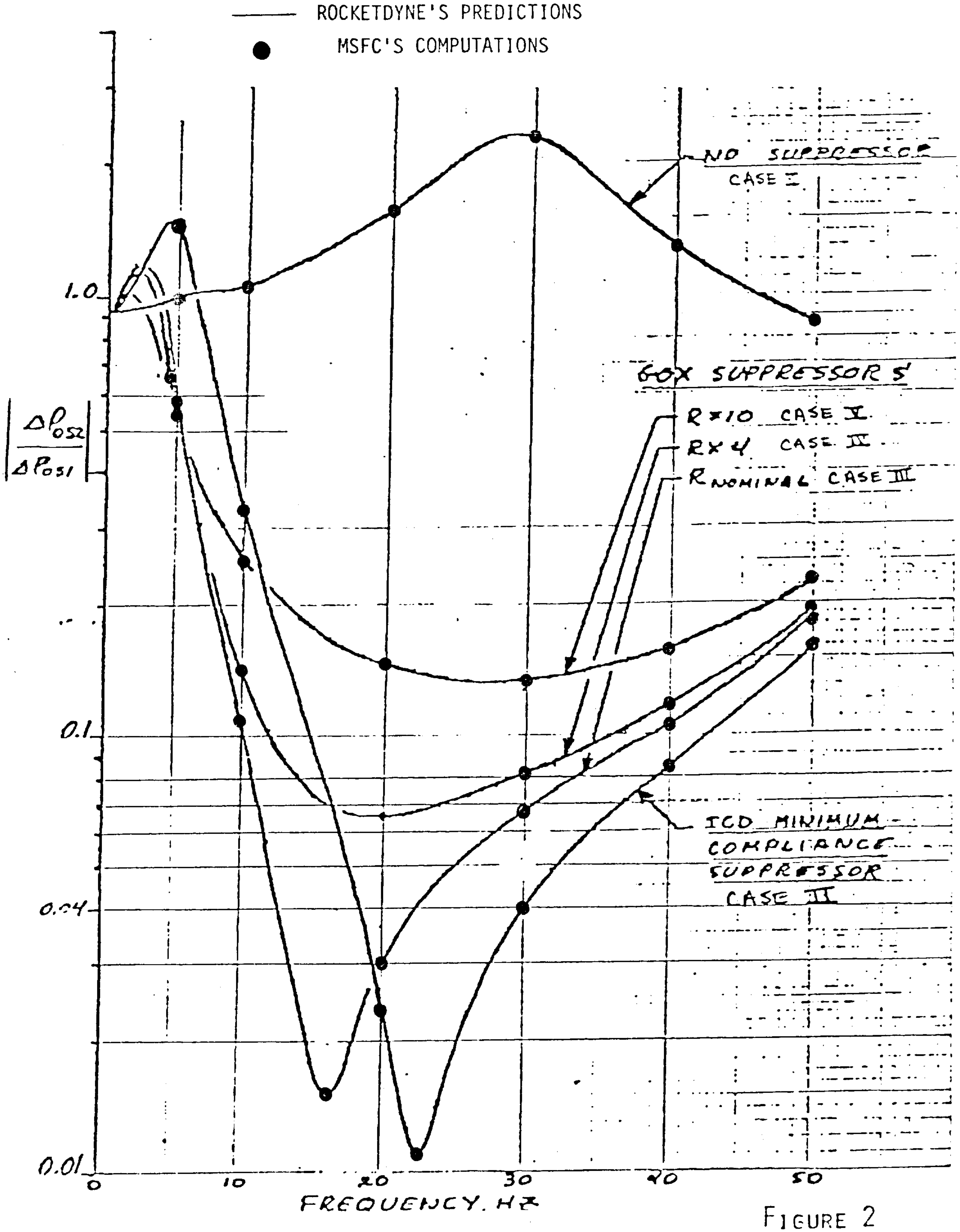


FIGURE 2

APPENDIX 1

POGO EQUATIONS AS OBTAINED FROM ROCKETDYNE

(1) $\left(36.45 + S\right) \Delta N_1 = 119.6 \; \Delta \dot{W}_{OT1}$

(2) $\left(45.7\right) \Delta \dot{W}_{OT1} = \Delta P_{OD2} - \Delta P_{OD1} - 2.625 \; \Delta N_1$

(3) $\Delta P_{OD1} = \Delta P_{OS1} + 1.872 \; \Delta N_1 - \left[0.3958 + 3.12^{(-3} S \right] \left[\Delta \dot{W}_{OD1} - \Delta \dot{W}_{OT1} \right]$

(4)
$$\Delta P_{OD2} = \frac{\left\{ 0.167\, S \left(1 + \frac{S}{14.6}\right) \left[1 + \frac{1.82\, S}{109} + \frac{S^2}{(109)^2} \right] - 0.672\, S \left(1 + \frac{S}{22.6}\right) \left[1 + \frac{1.89\, S}{124} + \frac{S^2}{(124)^2} \right] \Delta \dot{W}_{OT1} \right\}}{\left\{ \left(1 + \frac{S}{3.4}\right) \left(1 + \frac{S}{225}\right) \left[1 + \frac{1.91\, S}{37.2} + \frac{S^2}{(37.2)^2} \right] \right\}}$$

(5)
$$\Delta \dot{W}_{OD2} = \frac{0.026\, S \left(1 + \frac{S}{17.8}\right)\left(1 + \frac{S}{71.4}\right)}{\left(1 + \frac{S}{3.3}\right)\left(1 + \frac{S}{35}\right)\left(1 + \frac{S}{49.9}\right)\left(1 + \frac{S}{490}\right)} + \frac{\left(1 + \frac{S}{9.4}\right)\left(1 + \frac{S}{14.8}\right)\left(1 + \frac{S}{80.4}\right)\left(1 + \frac{S}{189}\right)}{\left(1 + \frac{S}{4.9}\right)\left(1 + \frac{S}{18.4}\right)\left(1 + \frac{S}{46.8}\right)\left(1 + \frac{S}{260}\right)} \; \Delta \dot{W}_{OT1}$$

(6) $\Delta P_{OD1} = \left(2.294\right) \Delta \dot{W}_{OD1} + e^{-0.00543\, S} \left[\Delta P^{*}_{OS2} - 2.294 \; \Delta \dot{W}^{*}_{OS2} \right]$

(7) $\Delta P^{*}_{OS2} = -\left(2.294\right) \Delta \dot{W}^{*}_{OS2} + e^{-0.00543\, S} \left[\Delta P_{OD1} + 2.294 \; \Delta \dot{W}_{OD1} \right]$

(8) $\Delta P^{*}_{OS2} - 1 = L^{*}_{OS}\, S \left[\dot{W}^{*}_{OS2} - \Delta \dot{W}_A \right]$

(9) $C_{OS}\, S = \Delta \dot{W}^{*}_{OS2} - \Delta \dot{W}_A - \Delta \dot{W}_{OD2}$

(10) $\Delta P^{*}_{OS2} - \Delta P_A = \left(R_A + L_{AS}\right) \Delta \dot{W}_A$

$$(11) \quad \Delta P_A \, C_A \left(s^2 + 0.8635\, s + 0.38014 \right) = \left(s + 3.878 \right) \Delta \dot{W}_A$$

$$(12) \quad \Delta P_c = \frac{\left\{ 0.073\, s \left(1 + \frac{s}{15.5} \right) \left(1 + \frac{s}{63} \right) - 0.327\, s \left(1 + \frac{s}{16.4} \right) \left(1 + \frac{s}{62.9} \right) \Delta \dot{W}_{OT1} \right\}}{\left\{ \left(1 + \frac{s}{3.45} \right) \left(1 + \frac{s}{434} \right) \left[1 + \frac{1.64\, s}{30.2} + \frac{s^2}{(30.2)^2} \right] \right\}}$$

APPENDIX 2

NONDIMENSIONAL VERSION OF POGO EQUATIONS

(1) $(36.45+s)\, X_6 = 119.6\, X_7$

(2) $45.7\, \psi_7 = \psi_8 - \psi_3 - 2.625\, \psi_6$

(3) $\psi_3 = \psi_2 + 1.872\, \psi_6 - \left[.3958 + 3.12^{-3} s\right]\left[\psi_1 - \psi_7\right]$

(4) $$\psi_8 = \frac{\left\{ .167 s\left(1+\frac{s}{14.6}\right)\left[1 + \frac{1.82 s}{109} + \frac{s^2}{(109)^2}\right] - .672 s\left(1+\frac{s}{22.6}\right)\left[1+\frac{1.89 s}{124} + \frac{s^2}{(124)^2}\right] \psi_7 \right\}}{\left\{ \left(1+\frac{s}{3.4}\right)\left(1+\frac{s}{225}\right)\left[1 + \frac{1.91 s}{37.2} + \frac{s^2}{(37.2)^2}\right]\right\}}$$

(5) $$\psi_4 = \frac{.026 s\left(1+\frac{s}{17.8}\right)\left(1+\frac{s}{71.4}\right)}{\left(1+\frac{s}{3.3}\right)\left(1+\frac{s}{35}\right)\left(1+\frac{s}{49.4}\right)\left(1+\frac{s}{490}\right)} + \frac{\left(1+\frac{s}{9.4}\right)\left(1+\frac{s}{14.8}\right)\left(1+\frac{s}{80.4}\right)\left(1+\frac{s}{189}\right)}{\left(1+\frac{s}{4.4}\right)\left(1+\frac{s}{18.4}\right)\left(1+\frac{s}{86.8}\right)\left(1+\frac{s}{260}\right)}\, \psi_7$$

(6) $\psi_3 = 2.294\, \psi_1 + e^{-.00543 s}\left[\psi_9 - 2.294\, \psi_{10}\right]$

(7) $\psi_9 = -(2.294)\, \psi_{10} + e^{-.00543 s}\left[\psi_3 + 2.294\, \psi_1\right]$

(8) $\psi_9 - 1 = l_{os}^{*}\, s\left[\psi_{10} - \psi_5\right]$

(9) $C_{S2}\, s = \psi_{10} - \psi_5 - \psi_4$

(10) $\psi_9 - \psi_{12} = (R_A + L_A s)\, \psi_5$

(11) $\psi_{12}\, C_A \left[s^2 + .8635 s + .38014 \right] = (s + 3.878)\, \psi_5$

(12) $$\psi_{11} = \frac{\left\{ .073 s \left(1 + \frac{s}{15.5}\right)\left(1 + \frac{s}{63}\right) - .327 s \left(1 + \frac{s}{16.4}\right)\left(1 + \frac{s}{62.4}\right) \psi_7 \right\}}{\left\{ \left(1 + \frac{s}{3.45}\right)\left(1 + \frac{s}{434}\right)\left[1 + \frac{1.64 s}{30.2} + \frac{s^2}{(30.2)^2}\right] \right\}}$$

$\frac{\Delta \dot{W}_{OO1}}{\Delta P_{OS2}} = X_1$, $\frac{\Delta P_{OS1}}{\Delta P_{OS2}} = X_2$, $\frac{\Delta P_{OD1}}{\Delta P_{OS2}} = X_3$, $\frac{\Delta \dot{W}_{OO2}}{\Delta P_{OS2}} = X_4$, $\frac{\Delta \dot{W}_A}{\Delta P_{OS2}} = X_5$, $\frac{\Delta N_1}{\Delta P_{OS2}} = X_6$,

$\frac{\Delta \dot{W}_{OT1}}{\Delta P_{OS2}} = X_7$, $\frac{\Delta P_{OD2}}{\Delta P_{OS2}} = X_8$, $\frac{\Delta P^*_{OS2}}{\Delta P_{OS2}} = X_9$, $\frac{\Delta \dot{W}^*_{OS2}}{\Delta P_{OS2}} = X_{10}$, $\frac{\Delta P_C}{\Delta P_{OS2}} = X_{11}$, $\frac{\Delta P_A}{\Delta P_{OS2}} = X_{12}$

APPENDIX 3

EQUATIONS IN POLYNOMIALS OF S

(1) $(36.45 + s)X_6 - (119.6)X_7 = 0$

(2) $(1.0)X_3 + (2.625)X_6 + (45.7)X_7 - (1.0)X_8 = 0$

(3) $(0.3958 + 3.12^{\backslash -3}s)X_1 - (1.0)X_2 + (1.0)X_3 - (1.872)X_6 - (0.3958 + 3.12^{\backslash -3}s)X_7 = 0$

(4) $(0.6725 + 3.99771^{\backslash -2}s^2 + 4.96913^{\backslash -4}s^3 + 1.93375^{\backslash -6}s^4)X_7 + (1.0 + 3.49908^{\backslash -1}s + 1.73593^{\backslash -2}s^2$

$+ 2.82867^{\backslash -4}s^3 + 9.44611^{\backslash -7}s^4)X_8 - (0.167s + 1.42268^{\backslash -2}s^2 + 2.05045^{\backslash -4}s^3 + 9.62747^{\backslash -7}s^4) = 0$

(5) $(1 + 0.65089s + 1.3791^{\backslash -1}s^2 + 1.1105^{\backslash -2}s^3 + 4.0291^{\backslash -4}s^4 + 7.0099^{\backslash -6}s^5 + 5.6584^{\backslash -8}s^6 + 1.8544^{\backslash -10}s^7 + 1.9494^{\backslash -13}s^8)X_4$

$-(1 + 5.4556^{\backslash -1}s + 9.4258^{\backslash -2}s^2 + 7.0877^{\backslash -3}s^3 + 2.5591^{\backslash -4}s^4 + 4.6065^{\backslash -6}s^5 + 4.0^{\backslash -8}s^6 + 1.4742^{\backslash -10}s^7 + 1.6920^{\backslash -13}s^8)X_7$

$-(0.026s + 9.54656^{\backslash -3}s^2 + 9.97209^{\backslash -4}s^3 + 4.18514^{\backslash -5}s^4 + 7.25467^{\backslash -7}s^5 + 5.13687^{\backslash -9}s^6 + 1.11962^{\backslash -11}s^7) = 0$

(6) $(2.294)X_1 - (1.0)X_3 + (1 - 5.43^{\backslash -3}s + 1.47425^{\backslash -5}s^2 - 2.6683^{\backslash -8}s^3 + 3.6223^{\backslash -11}s^4 - 3.93385^{\backslash -14}s^5$

$+ 3.56014^{\backslash -17}s^6 - 2.76165^{\backslash -20}s^7 + 1.87447^{\backslash -23}s^8 - 1.13093^{\backslash -26}s^9)X_9 + (-2.294 + 1.24564^{\backslash -2}s$

$- 3.38193^{\backslash -5}s^2 + 6.12108^{\backslash -8}s^3 - 8.30956^{\backslash -11}s^4 + 9.02425^{\backslash -14}s^5 - 8.16696^{\backslash -11}s^6 + 6.33523^{\backslash -20}s^7$

$- 4.30^{\backslash -23}s^8 + 2.59435^{\backslash -26}s^9)X_{10} = 0$

$$(7)\quad \Big(2.294 - 1.24564\backslash^{-2}\, S + 3.38193\backslash^{-5}\, S^2 - 6.12108\backslash^{-8}\, S^3 + 8.30956\backslash^{-11}\, S^4 - 9.02425\backslash^{-14}\, S^5 + 8.16696\backslash^{-17}\, S^6$$

$$- 6.33529\backslash^{-20}\, S^7 + 4.30\backslash^{-23}\, S^8 - 2.59435\backslash^{-26}\, S^9\Big) X_1 + \Big(1 - 5.43\backslash^{-3}\, S + 1.47425\backslash^{-5}\, S^2 - 2.6683\backslash^{-8}\, S^3$$

$$+ 3.6223\backslash^{-11}\, S^4 - 3.93385\backslash^{-14}\, S^5 + 3.56014\backslash^{-17}\, S^6 - 2.76165\backslash^{-20}\, S^7 + 1.87447\backslash^{-23}\, S^8 - 1.13093\backslash^{-26}\, S^9\Big) X_3$$

$$- (1.0)\, X_9 - (2.294)\, X_{10} = 0$$

$$(8)\quad (L^*_{os}\, S)\, X_5 + (1.0)\, X_9 - (L^*_{os}\, S)\, X_{10} - 1.0 = 0$$

$$(9)\quad -(1.0)\, X_4 - (1.0)\, X_5 + (1.0)\, X_{10} - C_{sz}\, S = 0$$

$$(10)\quad -(R_A + L_A\, S)\, X_5 + (1.0)\, X_9 - (1.0)\, X_{12} = 0$$

$$(11)\quad -(3.878 + S)\, X_5 + (0.38014 + 0.8635\, S + S^2)\, C_A\, X_{12} = 0$$

$$(12)\quad \Big(0.327\, S + 2.51794\backslash^{-2}\, S^2 + 3.19534\backslash^{-4}\, S^3\Big) X_7 + \Big(1 + 3.4646\backslash^{-1}\, S + 1.76299\backslash^{-2}\, S^2 + 3.5660\backslash^{-4}\, S^3$$

$$+ 7.32278\backslash^{-7}\, S^4\Big) X_{11} - \Big(0.073\, S + 5.86841\backslash^{-3}\, S^2 + 7.47568\backslash^{-5}\, S^3\Big) = 0$$